[3] Coles, D. E., "The Turbulent Boundary Layer in a Compressible Fluid," R-403-PR, Sept. 1962, The Rand Corp., Santa Monica, Calif.

[4] Economos, C., "A Transformation Theory for the Compressible Turbulent Boundary Layer with Mass Transfer," *AIAA Journal*, Vol. 8, No. 4, April 1970, pp. 758–764.

[5] Morkovin, M. W., "Effects of Compressibility on Turbulent Flow," International Symposium on the Mechanique de la Turbulence, 1962, Centre National de la Recherche Scientifique, Paris, pp. 367–380.

[6] Tennekes, H., "Law of the Wall for Turbulent Boundary Layers in Compressible Flow," *AIAA Journal*, Vol. 5, No. 3, March 1967, pp. 489–492.

[7] Scott, C. T. et al., "Measurements of Velocity and Concentration Profiles for Helium Injection Into a Turbulent Boundary Layer Flowing Over an Axial Circular Cyclinder," Part I—"Experimental Results," HTL-TR-55, Feb. 1964, University of Minnesota, Minneapolis, Minn.

[8] Coles, D. E., "Measurements in the Boundary Layer on a Smooth Flat Plate in Supersonic Flow, III. Measurements in a Flat Plate Boundary Layer at the Jet Propulsion Lab.," Rept. 20-71, June 1953, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, Calif.

[9] Winkler, E. M. and Cha, M. H., "Investigation of Flat Plate Hypersonic Turbulent Boundary Layers with Heat Transfer at a Mach Number of 5.2," Rept. 6631, Sept. 1959, U.S. Naval Ordnance Lab., White Oak, Md.

# Investigation of Turbulent Recovery Factor in Hypersonic Helium Flow

David H. Rudy* and Leonard M. Weinstein*
*NASA Langley Research Center, Hampton, Va.*

## Nomenclature

$M$ = Mach number
$r$ = recovery factor $(T_{aw} - T_e)/(T_o - T_e)$
$R$ = Reynolds number
$T$ = temperature

### *Subscripts*

$e$ = local conditions at edge of boundary layer
$aw$ = adiabatic wall
$o$ = total
$x$ = distance along plate from leading edge
$v$ = virtual origin (at peak of recovery factor in transition region)

An accurate knowledge of the temperature recovery factor is important in the reduction of wind-tunnel heat-transfer measurements to coefficient form, especially when those measurements are made at nearly adiabatic conditions. Numerous experimental measurements of the recovery factor have been made for laminar, transitional, and turbulent boundary layers in air for Mach numbers up to 5 (i.e., Refs. 1–9). A few measurements have also been made between Mach 5 and 6.[10–12] There has actually been no pressing need for precise air recovery factor measurements at Mach numbers above 5 because the total temperatures necessary to avoid liquefaction of air at these high Mach numbers place the usual wall temperatures far below the adiabatic values. However, in helium wind tunnels, hypersonic Mach numbers can be achieved without heating the flow and therefore an accurate value of the recovery factor is again needed for reduction of heat-transfer data. The only recovery factor study previously conducted in helium involved laminar and transitional flow at Mach 6.5.[13] The present investigation was therefore initiated to experimentally obtain the turbulent recovery factor in hypersonic helium flow.

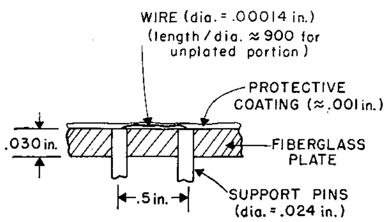


**Fig. 1 Lateral cut showing instrumentation for surface temperature measurement (not to scale).**

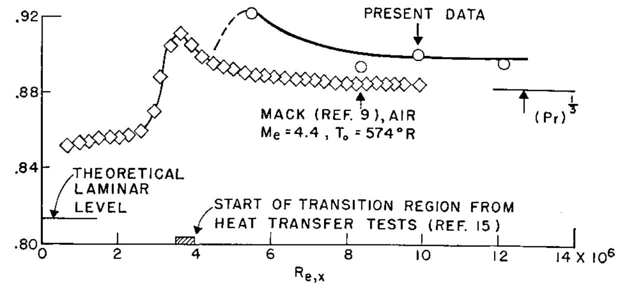


**Fig. 2 Recovery factor distribution for wedge model ($M_e$ = 6.8, helium).**

The present measurements of turbulent recovery factor were made in the Langley 22-in. Mach 20 helium tunnel using a thin fiber glass plate mounted on a sharp leading edge (0.002-in. thick), 10° half-angle wedge. The local Mach number $M_e$ on the 10° wedge was 6.8 with a negligible local pressure gradient.

The model was designed so that the fiber glass surface would be essentially insulated and thus, the equilibrium surface temperature would be very nearly the adiabatic-wall temperature. Longitudinal heat conduction was negligible because of the small cross section and low conductivity of the fiber glass plate. Calculations also indicated negligible error as the result of radiation from the tunnel wall. To accurately measure the surface temperature, the model was instrumented with thin wire (0.00014-in. diam) resistance thermometers, the ends of which were copper-plated for attachment to support pins, as shown in Fig. 1. A conservative analysis indicated that conduction through the steel support pins had only a slight (less than 0.1°R) effect on the wire temperature. Therefore, the wires were assumed to be correctly measuring the surface temperature. During the last half of the 93-sec

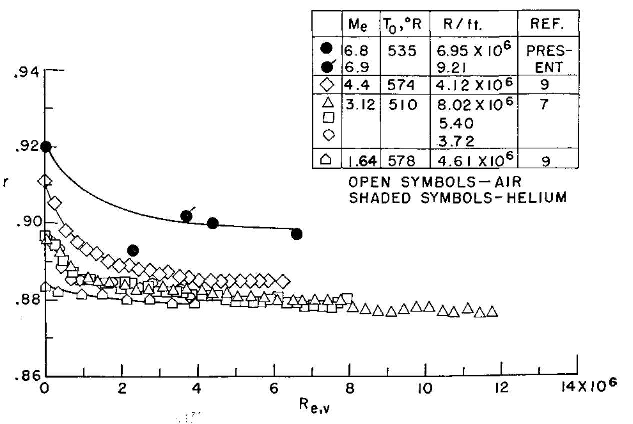


**Fig. 3 Variation of turbulent recovery factor with Reynolds number based on distance from recovery factor peak in transition region.**

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* Aerospace Engineers, Flow Analysis Section, Aero-Physics Division.

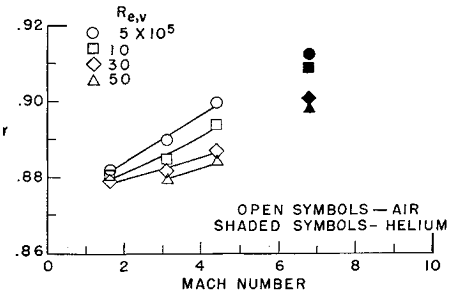


**Fig. 4 Effect of Mach number on turbulent recovery factor.**

test the ratio of the measured surface temperature to the total temperature changed less than 0.35% for each wire. Recovery factors were then calculated using the equilibrium surface temperature as adiabatic-wall temperature in the defining equation for recovery factor.

The results of the test are shown in Fig. 2. No data were obtained in the laminar region; however, the theoretical values for the helium recovery factor predicted by laminar similar solutions[14] at Mach 6.8 is indicated. The approximate beginning of the transition region determined from heat-transfer tests with unheated flow[15] using a thin metal plate mounted on the same wedge model at similar test conditions is also shown. In Ref. 15 the end of the transition region was found to be located at a distance approximately twice as far from the leading-edge as the beginning. Based on these Ref. 15 results, the first data point is believed to be in the transition region near the location of the peak in recovery factor. This trend is consistent with previous air measurements such as the data of Mack[9] shown in Fig. 2 for comparison. Based on the fairing through the data, the turbulent helium recovery factor at high Reynolds numbers and Mach 6.8 is approximately 0.899 $\pm$ 0.002. This is significantly higher than the generally used value of 0.883 (cube root of the molecular Prandtl number).

To investigate the effect of Reynolds number and Mach number on the turbulent recovery factor previous air measurements were examined. Fig. 3 shows, for several Mach numbers, the measured recovery factor as a function of Reynolds number based on distance from the location of the peak values in the transition region. As shown in Fig. 3, all the data for each Mach number follow the same trend, with the recovery factor dropping rapidly from a peak value to a slightly decreasing value at higher Reynolds numbers.

The data at several Reynolds numbers are cross plotted in Fig. 4 as a function of Mach number. In contrast to the theory of Tucker and Maslin,[16] the data indicate that the turbulent recovery factor increases with increasing Mach number.

### References

[1] Brinich, P. F., "Recovery Temperature, Transition, and Heat Transfer Measurements at Mach 5," TN D-1047, Aug. 1961, NASA.

[2] Dmohoski, S. M., "Temperature Recovery Factors on a 40-Degree Cone Cylinder with Turbulent Boundary Layer at Mach Numbers 4.1 and 4.8," NAVORD Rept. 5763, Aug. 1957, U.S. Naval Ordnance Lab., White Oak, Md.

[3] Jack, J. R., and Moskowitz, B., "Experimental Investigation of Temperature Recovery Factors on a 10° Cone at Angle of Attack at a Mach Number of 3.12," TN 3256, July 1954, NACA.

[4] Shoulberg, R. H., Hill, J. A. F., and Rivas, J. M. A., Jr., "An Experimental Determination of Flat Plate Recovery Factors for Mach Numbers Between 1.90 and 3.14," M.I.T. Wind Tunnel Rept. 36, May 1952, M.I.T., Cambridge, Mass.

[5] Stine, H. A., and Scherrer, R., "Experimental Investigation of the Turbulent Boundary Layer Temperature-Recovery Factor on Bodies of Revolution at Mach Numbers from 2.0 to 3.8," TN-2664, March 1952, NACA.

[6] Stalder, J. R., Rubesin, M. W., and Tendeland, T., "A Determination of the Laminar-, Transitional-, and Turbulent-Boundary Layer Temperature Recovery Factors on a Flat Plate in Supersonic Flow," TN-2077, June 1950, NACA.

[7] Brinich, P. F., "Boundary Layer Transition at Mach 3.12 With and Without Single Roughness Elements," TN 3267, 1954, NACA.

[8] Evvard, J. C., Tucker, M., and Burgess, W. C., Jr., "Statistical Study of Transition-Point Fluctuations in Supersonic Flow," TN 3100 1954, NACA.

[9] Mack, L. M., "An Experimental Investigation of the Temperature Recovery Factor," Rept. 20-80, Aug. 1954, Jet Propulsion Lab., Pasadena, Calif.

[10] Tendeland, T., "Effects of Mach Number and Wall-Temperature Ratio on Turbulent Heat Transfer at Mach Numbers from 3 to 5," TR R-16, 1959, NASA.

[11] DeLauer, R. D. and Nagamatsu, H. T., "Experimental Heat Transfer at Hypersonic Mach Numbers," Memo 14, April 1953, California Institute of Technology, Guggenheim Aeronautical Lab., Pasadena, Calif.

[12] Reller, J. O., Jr. and Hamaker, Frank M., "Temperature Recovery Factors on a Slender 12° Cone Cylinder at Mach Numbers from 3.0 to 6.3 and Angles of Attack up to 45°," R&M A55G20, Oct. 1955, NACA.

[13] Nicoll, K. M., "An Experimental Investigation of Laminar Hypersonic Cavity Flows," Part II: "Heat Transfer and Recovery Factor Measurements," ARL Rept. 63-73, Jan. 1964, Wright-Patterson Air Force Base, Ohio.

[14] Nicoll, K. M., "Investigation of the Laminar Boundary-Layer on a Flat Plate in Helium Using the Crocco Method," ARL Rept. 62-345, May 1962, Wright-Patterson Air Force Base, Ohio.

[15] Wagner, R. D., Jr., et al., "Influence of Measured Free-Stream Disturbances on Hypersonic Boundary Layer Transition," AIAA Paper 69-704, San Francisco, Calif., 1969.

[16] Tucker, M. and Maslen, S. H., "Turbulent Boundary-Layer Temperature Recovery Factors in Two-Dimensional Supersonic Flow," TN 2296, 1951, NACA.

# Nonlinear Elastic Response of an Incased Tubular Grain

ROGER J. EVANS*

AND

MICHAEL S. HAYCOCK†

*University of Washington, Seattle, Wash.*

### Introduction

THE mechanical response of highly loaded solid propellants is significantly nonlinear at strain levels of 5 or 10% in uniaxial stress states. A previous paper in this Journal[1] has shown that useful results may be obtained from the stress analysis of elastic solids that are physically nonlinear conditions of geometric linearity. Solutions were presented for the internal pressurization of a rigidly incased tubular grain and for thermal stresses in an incased grain.

Baltrukonis and Vaishnev[2] considered finite axisymmetric deformations with regard to the analysis of solid propellant grains. Their work, however, was restricted to a particular class of incompressible elastic solids.

The purpose of this Note is to extend the analysis of Pister and Evans[1] to the more realistic case: where the problem formulation takes into account both physical and geometric nonlinearity. Such an extension is of importance with regard both to extending the range of useful solutions and to justifying the simultaneous consideration of geometric linearity and physical nonlinearity. Solutions for geometric

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* Assistant Professor, Department of Civil Engineering.

† Graduate Student, Department of Civil Engineering.